

Putnam Exam Questions

1: Let S be a set of real numbers which is closed under multiplication (that is, if a and b are in S , then so is ab). Let T and U be disjoint subsets of S whose union is S . Given that the product of any three (not necessarily distinct) elements of T is in T and that the product of any three elements of U is in U , show that at least one of the two subsets T, U is closed under multiplication.

2: Let S be the set of ordered triples (a, b, c) of distinct elements of a finite set A . Suppose that

1 $(a, b, c) \in S$ if and only if $(b, c, a) \in S$.

2 $(a, b, c) \in S$ if and only if $(c, b, a) \notin S$

3 (a, b, c) and (c, d, a) are both in S if and only if (b, c, d) and (d, a, b) are both in S .

Prove that there exists a one-to-one function g from A to \mathbb{R} such that

$$g(a) < g(b) < g(c)$$

implies $(a, b, c) \in S$. Note: \mathbb{R} is the set of real numbers.