

Putnam Exam Questions

1: Find functions $f(x) : \mathbb{R} \rightarrow \mathbb{R}$ such that

1) $f^{-1}(x) = \frac{1}{f(x)}$

2) $f^{-1}(x) = -f(x)$.

Dr. Blair will talk about these during his Math club talk Wednesday at 3:00.

2: For what pairs (a, b) of positive real numbers does the improper integral

$$\int_b^{\infty} \left(\sqrt{\sqrt{x+a} - \sqrt{x}} - \sqrt{\sqrt{x} - \sqrt{x-b}} \right) dx$$

converge?

3: An ellipse whose semi-axes have lengths a and b , rolls without slipping on the curve $y = c \sin\left(\frac{x}{a}\right)$. How are a, b, c related, given that the ellipse completes one revolution when it traverses one period of the curve?